import numpy as np

import scipy.stats as sts

import matplotlib.pyplot as plt

# Grid of possible true hair darkness values (0 = blonde, 1 = black)

mu = np.linspace(0, 1, 300)

# Prior: Baby is expected to have dark hair, around 0.8 darkness with small uncertainty

prior = sts.norm.pdf(mu, loc=0.8, scale=0.1)

prior = prior / prior.sum()

# Plot the prior

plt.plot(mu, prior, label='Prior')

plt.xlabel("Baby's Hair Darkness (0 = Blonde, 1 = Black)")

plt.ylabel("Probability Density")

plt.title("Prior Belief About Baby's Hair Darkness")

plt.legend()

plt.show()

# Likelihood: Observation from photo = 0.5 (medium brown), lighting adds noise

# Measurement error std dev = 0.15

def likelihood\_func(observation, mu\_values):

likelihood = sts.norm.pdf(observation, loc=mu\_values, scale=0.15)

return likelihood / likelihood.sum()

# Compute the likelihood based on the photo

likelihood = likelihood\_func(0.5, mu)

# Plot the likelihood

plt.plot(mu, likelihood, label='Likelihood')

plt.xlabel("Baby's Hair Darkness (0 = Blonde, 1 = Black)")

plt.ylabel("Likelihood")

plt.title("Likelihood Given Observed Hair Darkness = 0.5")

plt.legend()

plt.show()

# Compute the unnormalized posterior

unnormalized\_posterior = prior \* likelihood

# Plot the unnormalized posterior

plt.plot(mu, unnormalized\_posterior, label='Unnormalized Posterior')

plt.xlabel("Baby's Hair Darkness (0 = Blonde, 1 = Black)")

plt.ylabel("Unnormalized Probability")

plt.title("Posterior Belief About Baby's Hair Color Darkness")

plt.legend()

plt.show()